Further Considerations and Analysis of Stereophonic Phenomena

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Introduction

The physical facts regarding destructive interference have been largely undiscussed over the course of six-plus decades of sound research, as it pertains to loudspeaker stereophony. In particular, destructive interference at the eardrums (tympani) of humans listening to driven stereo loudspeakers, by way of pan-potted voltage signal division, can be anticipated through the same sound-pressure analytic techniques of the Bauer method.[1] But it should be understood that a plurality of sound sources is needed for interference to take place, because a single sound source, unaided, cannot physically interfere with itself (in the manner to be described below).

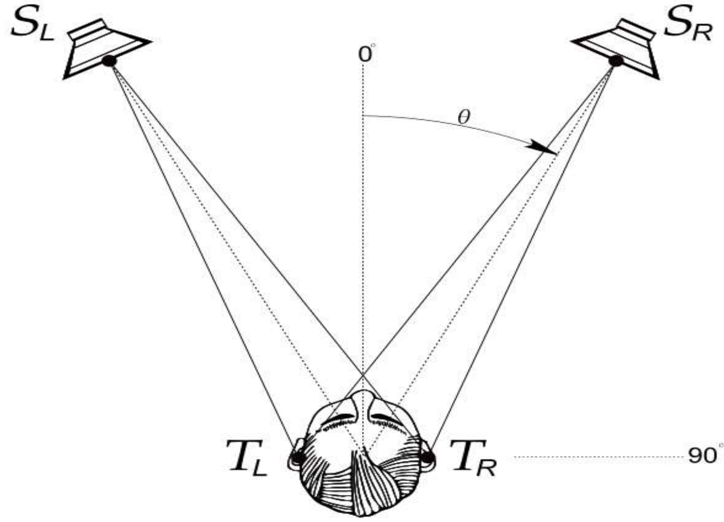

Figure 1

We begin by considering the geometry of Fig. 1, which portrays an idealized stereophonic array comprised of two laterally-balanced loudspeakers S_L and S_R, each providing airborne sound-pressure vectors impressing onto two tympanic membranes T_L and T_R of an idealized human head. As the sound pressures from each loudspeaker are summed at each tympanum, this sets up a condition of coincidental destructive interference (or, "CDI") at the membranes, given the differing lengths of the ipsilateral (*i.e.*, "direct") and contralateral ("cross") sound-pressure vectors from their respective loudspeakers. As a first approximation, the difference of lengths of these direct- and cross- feeds can be set forth with the following equations:

$$|(|S_RT_L| - |S_RT_R|)| = |T_LT_R| * \sin\theta, \text{ and} \quad (1)$$

$$|(|S_LT_R| - |S_LT_L|)| = |T_LT_R| * \sin\theta, \text{ where} \quad (2)$$

S_RT_L = the contralateral sound-pressure vector to the left ear,

S_LT_R = the contralateral sound-pressure vector to the right ear,

S_LT_L = the ipsilateral sound-pressure vector to the left ear,

S_RT_R = the ipsilateral sound-pressure vector to the right ear,

$|T_LT_R|$ = the lateral distance between the tympanic membranes of the human ears, and

θ = angle of incidence from the loudspeaker array to the center (origin) of the human head.

(Note that the right-hand expressions of equations (1) and (2) are identical; thus, the application of either formula will produce the same result.)

The expression $|T_LT_R|$ is slightly shorter than the width of the human head by virtue of the ear canals, and if we say it is the value of, say, seven inches (0.583 feet) and let θ be 30 compass degrees, we can, as an example, calculate (1) or (2) as

$$\sin(30) = 0.5, .583 * \sin(30) = .292 \text{ feet.}$$

The speed of sound is about 1130 ft/sec, and the contralaterally-generated delay for the CDI condition is therefore 0.292/1130 or 258.1 μsec.

It would be desireable to produce a viable simulation of the human ear's amplitude response to an auditory stimulus applied monaurally to the array of Fig. 1, realized by an appropriate digital audio editing program, and visualized in the family of curves depicted in Fig. 2 below. These curves have been generated by a logarithmic sine-wave audio sweep from 20Hz to 20kHz to show zero-to-peak amplitude, and then transferred to a vector image graphic editor program to provide an accurately shaped line drawing. Thus, trace ***A*** shows the initial constant amplitude of the given audio sweep.

The "comb-filter" effect shown in trace ***B*** results from adding trace ***A*** to itself with an additional delay element in accordance with the vector summation analysis of Fig. 1 at the tympanic membranes. (This analysis can be further bolstered with the substitution of white noise in place of the sinusoidal log sweep.) The fundamental notch at $\mathbf{f_1}$, as well as subsequent notches corresponding to the odd harmonics of the frequency of $\mathbf{f_1}$, is produced by the summation of two airborne sound pressure vectors whose difference in arrival times equals half the wavelength of the fundamental notch frequency.

Another "comb-filter" effect is shown in trace ***C*** where the polarity of one of the loudspeakers of Fig. 1 is reversed. As expected, the lowest frequencies appear to buck out at the eardrums; it is also noted that the zeroes ("nulls") shift to what were the even harmonics of the fundamental notch of trace ***B,*** showing inversion of the poles ("peaks") and zeroes when the loudspeakers of Fig. 1 are driven out of phase with each other.

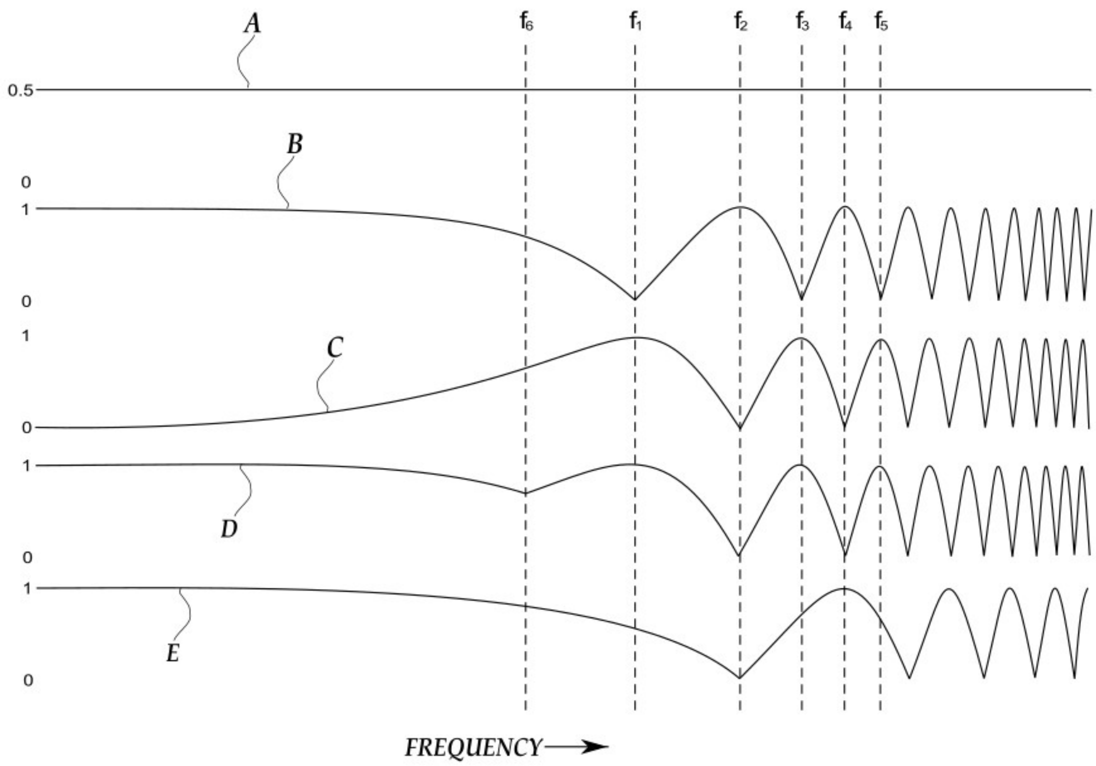

Figure 2

It can be seen in Fig. 2 that the fundamental notch of trace ***B*** (shown as at $\mathbf{f_1}$) has been moved, by an increase of one octave of frequency, to $\mathbf{f_2}$ when one of the loudspeakers of Fig. 1 is reversed in phase. If we were to propose some sort of circuitry to combine traces ***B*** and ***C*** in such a way to create a new hybrid trace (such as shown in trace ***D***), it may be possible to sidestep some of the interfering auditory mechanisms without introducing new variables into equations (1) and (2), or even adjusting the value of the existing sole parameter θ. To illustrate this, we compare the hybridized trace ***D*** with its closest non-hybridized analogue, trace ***E***, which has the same fundamental notch frequency ($\mathbf{f_2}$) as trace ***D***. We see that trace ***E*** has the same general form as trace ***B***, the key difference being that ***E***'s contralaterally-induced time delay is one half of that of ***B***. The hybridized trace ***D*** has, in effect, "emulated" the perception of a change in the incidental angle θ of Fig. 1 without a corresponding physical adjustment of the loudspeaker array, nor without a change in the position of the human head relative to the array, either of which, in turn, would affect angle θ. This is attributed directly to the hybridization effect just described.

We can now present an "emulation" function of θ, which we will call $F(\theta)$, such that

$$\sin(F(\theta)) = 0.5 * \sin\theta, \quad \text{or, solving for } F(\theta)\text{, we get} \qquad (3a)$$

$$F(\theta) = \arcsin(0.5 * \sin\theta), \qquad (3b)$$

and thus plot the emulated angle $F(\theta)$ as a function of the incidental angle θ, the sketch of which is shown in Fig. 3 as a solid curved line.

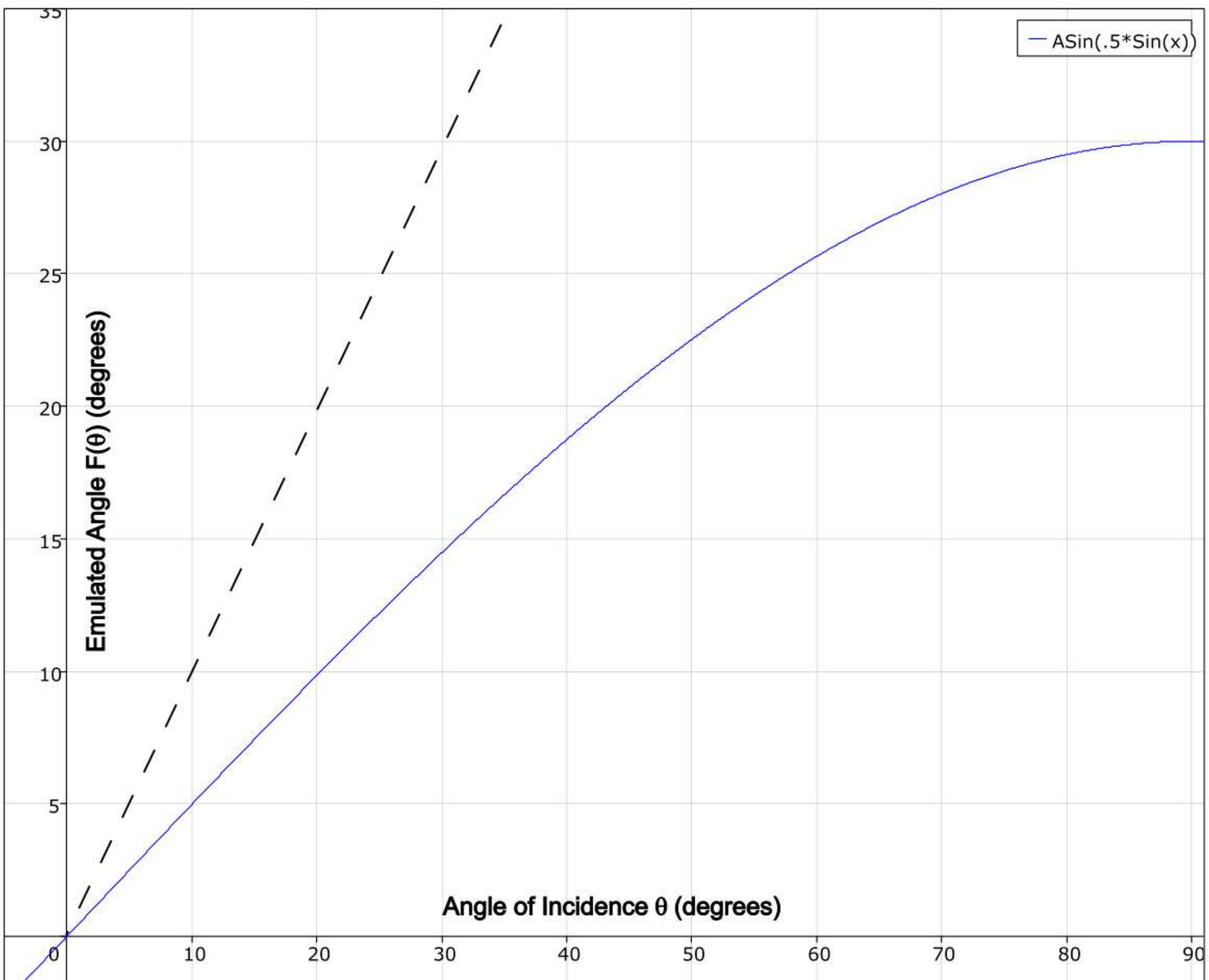

Figure 3

What is observed firsthand in Fig. 3 is a compression characteristic of the solid curved line

$F(\theta)$: as the physical angle θ increases to its full displacement of 90 degrees, $F(\theta)$ attains a maximum value of 30 degrees. The slope of the curve is not constant, but gradually diminishes as θ increases so that a 30 degree increase of θ (from 60 to 90 degrees) results in a mere 4.34 degree increase in the emulated, or witnessed, angle by the observer. Note that if the physical angle θ increases further from 90 degrees, then the observer will have crossed the proscenium between the loudspeakers of Fig. 1, into another adjoining quadrant of angular rotation, and the geometry of Fig. 3 will then reverse itself as a horizontal reflection of the solid curve.

The question arises about what is the straight dashed line shown in Fig. 3: this is simply the identity

$$F(\theta) = \arcsin(\sin\theta) = \theta,$$

which results from a raw signal with no hybridization being applied to it, and its resulting witnessed angle. In contrast to the aforementioned hybridized solid curved line, this linear straight dashed line exhibits no compression characteristic, even if the upper vertical limit of the sketch of Fig. 3 were to be extended without bounds. We can therefore say, in the non-hybridized instance, that the emulated angle $F(\theta)$, as witnessed by the observer, is the same as the incidental angle θ of Fig. 1.

Upon applying the principles given heretofore, it should then be possible to explain and validate some additional, previously unpublished observations. In the case of the observer of Fig. 1 maintaining a balanced azimuthal angle ("zero degrees azimuth"), while approaching the proscenium between the loudspeakers (see Fig. 1), the incidental angle θ increases to a limit of 90 degrees. However, in the case of the hybridized treatment of the auditioned signal, the perceived angle $F(\theta)$ approaches the limit of 30 degrees (see Fig. 3). As these limits are approached, the rate of change of $F(\theta)$ reduces to zero; however, due to the inverse-distance law, the human ear simultaneously detects an increase in loudness, thus reinforcing said limit of the perceived angle $F(\theta)$. *This reinforcement results in the observer being able to sense distance between himself and the perceived source of sound.*

In the case of the non-hybridized (raw) signal being applied to the loudspeakers of Fig. 1, we see that the reinforcement is compromised due to the failure of the human auditory system to perceive a limit of the angle $F(\theta)$. Even though the inverse-distance law is still noticeable, this serves to add not to the reinforcement but rather to the confusion of the listener, as when he draws nearer to the proscenium between the loudspeakers, the perceived source of sound does not draw nearer to him, which is in contradiction to the desired effect of inverse-distance law. The observer, in this case, then resorts to finding an equilibrium of distance to the angle of incidence θ, which is, in effect, locating his optimum position by finding a "sweet spot" in the sound field image lying between the loudspeakers. This does not mean to imply that the "sweet spot" protocol is without merit (as it is both widely-used and well-established), but on the other hand there is a decided disadvantage in that it allows for only *a single observer* to physically occupy the space of the sweet spot in any given instance.

Conclusion

The concept of destructive interference as it applies to two-loudspeaker stereophony was anticipated and established. The sound-pressure vectors requisite for the given CDI condition to exist were recognized, and defined mathematically. Through visualization by graphic means, the need for simulation of the human auditory response was realized. The concept of hybridizing an auditory signal was presented. The formula for an auditory function of "emulation" was discovered and propounded. A sketch of such a formula was then plotted and presented. A discussion on auditory reinforcement, or lack thereof, as a result of hybridizing ensued.

References

[1] B. B. Bauer, "Phasor Analysis of Some Stereophonic Phenomena," *J. Acoust. Soc. Am.*, vol. 33, pp. 1536-1539 (1961).